

# The BMW Model

## A new Framework for Teaching Macroeconomics

### Monetary and Fiscal Policy Interaction in a Closed Economy

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#### **Abstract**

In previous papers we developed a new framework for macroeconomic teaching-the BMW model- that can easily deal with issues like inflation targeting, monetary policy rules and central bank credibility. In this paper we use this basic framework and extend it for fiscal policy in a closed economy. Hence we potentially aim at describing fiscal and monetary policy interaction for a large closed economy like the USA. Relying on Nash equilibrium we model the nominal interest rate trap and the investment trap as prime examples for monetary and fiscal policy interaction. Additionally we will highlight the strategic interaction between the central bank and fiscal authorities in ordinary times. Last but not least we look at what happens if fiscal policy is overly ambitious. In {Bofinger & Mayer 2003 223 /id} we extend this basic framework and carry it over to a monetary union potentially describing EMU to capture the more complex interaction in a monetary union.

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|          |  |                  |
|----------|--|------------------|
| <b>1</b> | <b>Introduction .....</b>  | <b>1</b>         |
| <b>2</b> | <b>BMW: Its main building blocs.....</b>   | <b>1</b>         |
| <b>3</b> | <b>Modelling the strategic interaction between the government and the central bank 6</b>   |                  |
|          | <i>3.1 The reaction function of the central bank: analytically and graphically .....</i>   | <i>6</i>         |
|          | <i>3.2 The reaction function of the government: analytically and graphically.....</i>  | <i>7</i>         |
| <b>4</b> | <b>Application I: Nominal interest rate trap and investment trap as prime examples for coordinated macroeconomic interaction .....</b> | <b>9</b>         |
|          | <i>4.1 A nominal interest rate trap .....</i>  | <i>9</i>         |
|          | <i>4.2 Investment Trap.....</i>  | <i>12</i>        |
| <b>5</b> | <b>Application II: A Clear Assignment on the target level does not lead to a clear assignment on the instrument level.....</b>         | <b>15</b>        |
| <b>6</b> | <b>Application III: Modelling the strategic interaction between the central bank and the government in ordinary times.....</b>         | <b>17</b>        |
|          | <i>6.1 The Nash equilibrium: analytically and graphically .....</i>  | <i>17</i>        |
|          | <i>6.2 Comparative static analysis of the optimal solution.....</i>  | <i>19</i>        |
| <b>7</b> | <b>Application III: An overly ambitious central bank .....</b>   | <b>21</b>        |
| <b>8</b> | <b>Conclusions .....</b>   | <b>24</b>        |
|          | <b><i>APPENDIX.....</i></b>  | <b><i>25</i></b> |
|          | <i>Interest rate smoothing .....</i>   | <i>25</i>        |

## 1 Introduction

The aim of this paper is to extend the original BMW framework which is a static version of a New Keynesian Macro Model as developed by (Clarida, Gali, and Gertler 1999) and built in fiscal policy (Bofinger, Mayer, and Wollmershäuser 2002). So far we only considered monetary policy as the only macroeconomic agent that stabilizes shocks. Of course it is a fact that in everyday life we do not only observe the institution of the central bank, but also fiscal authorities. So it goes without saying that one should have a natural interest in modelling the strategic interaction between these two agents. In teaching literature, it is well documented that in particular in situations where the economy is hit by a large demand shock, or in situations where the economy is stuck in an investment trap a coordinated action of both macroeconomic agents or only of fiscal policy alone is a necessary precondition to restore full employment {Bofinger 2003 218 /id}. Besides these two extreme cases, which are well established we also aim at modelling monetary and fiscal policy interaction in ordinary times, when both agents are not restricted in using their instrument. In the Barro Gordon literature (1983) it is generally assumed that monetary policy is overly ambitious, trying to overcome monopolistic distortions in goods and labour markets by inflating the economy. We hold it as more plausible to assume that fiscal policy exhibits this bias. This assumption seems realistic as it is a stylised fact that fiscal policy has accumulated ever rising debt burdens over the past decades. In order to win the electorate fiscal policy tries to push output above its potential. As we will see such a scenario can potentially explain an unsustainable fiscal policy.

In {Bofinger & Mayer 2003 223 /id} we extend this set up to a monetary union in order to describe the specific problems in the game between fiscal and monetary policy interaction in EMU. The basic focus of this paper is to describe the fiscal and monetary policy interaction in a large closed economy like the USA.

## 2 BMW: Its main building blocs

In this section we introduce fiscal policy in the BMW framework. The BMW model allows a simple discussion of the game between the central bank and the government for a closed economy like the US. We assume that fiscal authorities are guided by a loss function. The focus will be on analysing the strategic interaction between the government and monetary policy in the

light of conflicting targets. We model the strategic interaction by means of a *Nash-equilibrium*. On a formal level we are faced with two players- the central bank and fiscal authorities. The government chooses some  $g$ , which is distributed from minus to plus infinity. The central bank chooses some  $r$ , which is distributed from null to plus infinity. We have a Nash equilibrium if for each player  $g^*/r^*$  is the first best response to the optimal strategy of the other player  $r^*/g^*$  so that

$$(1) \quad L_{CB}(g^*, r^*) \leq L_{CB}(g^*, r) \text{ and } L_G(g^*, r^*) \leq L_G(g, r^*)$$

holds (see (Gibbons 1992)).

The extended BMW model consists of four building blocs:

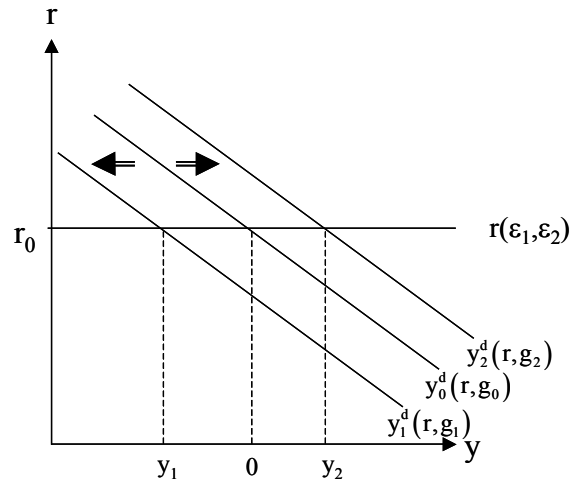
- an aggregate demand equation
- a Phillips curve equation, also labelled inflation adjustment equation
- the assumption that output is demand determined in the short run
- two loss functions depicting the way according to which monetary and fiscal policy is conducted.

*Aggregate demand*, which is presented, in the form of the output gap  $y$  depends on autonomous demand components  $a$ , negatively on the real interest rate and additionally as an innovation compared to the previous sections on the stance of fiscal policy  $g$ . The stance of fiscal policy is defined as expenditures minus revenues. Hence if  $g > 0$  the fiscal stance is expansionary, if  $g < 0$  the fiscal stance is contractionary.

$$(2) \quad y = a - br + \kappa g + \varepsilon_1$$

Chart 1 depicts that the instrument of fiscal policy is a shift parameter in the  $(y, r)$ -space. Starting with  $g_0$ , which corresponds to an output gap of zero an expansionary fiscal policy shifts the aggregate demand curve from  $y_0^d$  to  $y_2^d$  and -for a given level of interest rates  $r_0$ - the output gap will increase from 0 to  $y_2$ . A contractionary fiscal stance  $g_1$  shifts the aggregate demand curve from  $y_0^d$  to  $y_1^d$  for a given level of interest rates  $r_0$ .

**Chart 1: Fiscal and monetary policy interaction in the (y;r)-space**



The supply side of the economy is modelled by an *expectations-augmented Phillips curve*, which is also frequently labelled as inflation adjustment line (see (Walsh 2002)):

$$(3) \quad \pi = \pi_0 + dy + \varepsilon_2$$

Equation (3) can be derived as follows: Each period all firms negotiate new wage contracts for one period. Workers are assumed to care about the current state of economic activity  $y$  as well as on the expected inflation rate  $\pi^e$  over the life of the contract. For the sake of simplicity we assume that monetary policy is credible ( $\pi^e = \pi_0$ ). The nominal change in wages is then given by:

$$(4) \quad \Delta w = \pi_0 + dy$$

As firms are assumed to be monopolistic competitors which price their output at a constant mark up over marginal costs, mark-up pricing translates wage inflation into price inflation.

$$(5) \quad \pi = \Delta w + mu$$

Let us assume that the mark-up factor is equal to zero ( $mu=0$ ). By inserting equation (4) into (5) we get a static version of the Phillips curve as described by equation (3).

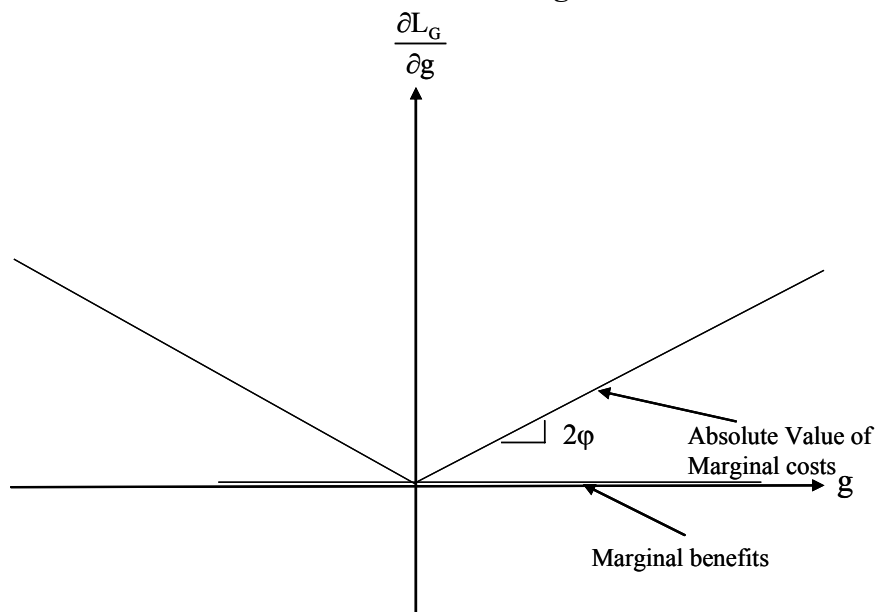
The *loss function* of the government is specified by equation (6):

$$(6) \quad L_G = \beta y^2 + \varphi g^{2^1}$$

Besides stabilizing squared deviations of the output gap around its potential we assume that policymakers are interested in stabilizing squared deviations of its instrument  $g$  around a long run sustainable level of  $g=0$ . Such a behaviour might be motivated for instance by the Treaty of Maastricht that penalizes excessive (downward) movements in the fiscal stance parameter  $g$ . Additionally, if  $g$  would be permanently larger than null the solution would exhibit some unpleasant arithmetic's as the fiscal balance would exhibit a structural deficit.

Loss function (6), which is commonly used in literature nests some interesting implications, which are rarely mentioned. The introduction of  $\varphi g^2$  puts a penalty on the use of the fiscal instrument, changes in  $g$  are- from the perspective of the government- not only associated with benefits (in the form of output stabilization) but also with costs (in terms of using the instrument).

**Chart 2: Absolute value of costs and benefits of using the fiscal instrument**



The marginal benefits of using the fiscal stance parameter are given by:

$$(7) \quad \frac{\partial L_G}{\partial g} = 2\beta\kappa(a - br + \kappa g + \varepsilon_1)$$

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<sup>1</sup> This loss function is very commonly used. See for instance (Leitemo 2003), (Andersen 2002), (Uhlig 2002).

Exploiting the envelope theorem equation (7) can equally be written as:

$$(8) \quad \left. \frac{\partial L_G}{\partial g} \right|_{r=\frac{a}{b}+\frac{1}{b}\varepsilon_1, g=0} = 0$$

Equation (8) depicts that the marginal benefits of changing the fiscal stance parameter are equal to null if monetary policy is conducted optimally.

The marginal costs of using the instrument are simply given by:

$$(9) \quad \frac{\partial L_G}{\partial g} = 2\varphi g$$

Chart 3 nicely depicts that in the absence of macroeconomic shocks to anticipate some results, marginal benefits will be equal to marginal costs from the perspective of fiscal policy if and only if  $g = 0$ . Note that the introduction of  $\varphi g^2$  as an independent goal of fiscal policy puts fiscal authorities in a comparatively weaker position compared to monetary policy. Therefore it will always be possible for the central bank to push its preferred bliss point through.

As an additional interesting case to consider we analyse an overly ambitious fiscal policy. Therefore we consider a modified loss function. We assume that fiscal policy tries to push output permanently above its potential in order to win the votes of the electorate.

$$(10) \quad L_G = \beta(y - k)^2 \quad k > 0$$

As we will see this loss function implies some unpleasant debt arithmetic's as the fiscal stance parameter  $g$  will be larger than null in equilibrium.

The loss function of the central bank- to complete the description of the basic equations- will be specified as in (Bofinger, Mayer, and Wollmershäuser 2002), to map the strategy of inflation targeting.

$$(11) \quad L_{CB} = \gamma(\pi - \pi_0)^2 + \lambda y^2$$

The central bank targets at stabilizing squared deviations of the inflation rate around the inflation target and squared deviations of the output gap around null. The parameters  $\gamma$  and  $\lambda$  depict the relative weights monetary policymakers put on the individual goal variables.

### **3 Modelling the strategic interaction between the government and the central bank**

Based on the concept of a Nash equilibrium we present two basic scenarios. First, a classical case where only a coordinated action of fiscal and monetary policy is a necessary precondition to restore a macroeconomic equilibrium, namely we will assume that we are stuck in an investment trap (Keynes 1936). Additionally we describe an overall economic environment that potentially describes the current situation in Japan. Hence the economy is hit by a large negative demand shock, such as a crash in stock markets. We will see that at best monetary policy- in such a deflationary environment- can set real interest rates equal to zero, which corresponds only to a partial stabilization of output. Therefore fiscal policy will have to step in to restore the bliss point ( $\pi_0 = \pi$ ,  $y=0$ ). Second, after having discussed these cases we will analyse the strategic interaction between fiscal and monetary policy in normal times. This means in particular that neither fiscal nor monetary policy faces an upper or lower bound for its instrument respectively. Additionally we will analyse the virtues of clear assignments on the target level and show that this does not lead to a unique assignment on the instrument level. Before considering these cases in detail in section 4 we will shortly develop the basic tools for analysing the strategic interaction between fiscal and monetary policy.

#### **3.1 The reaction function of the central bank: analytically and graphically**

Under all applications the central bank is assumed to solve the following (generalized) optimisation problem.

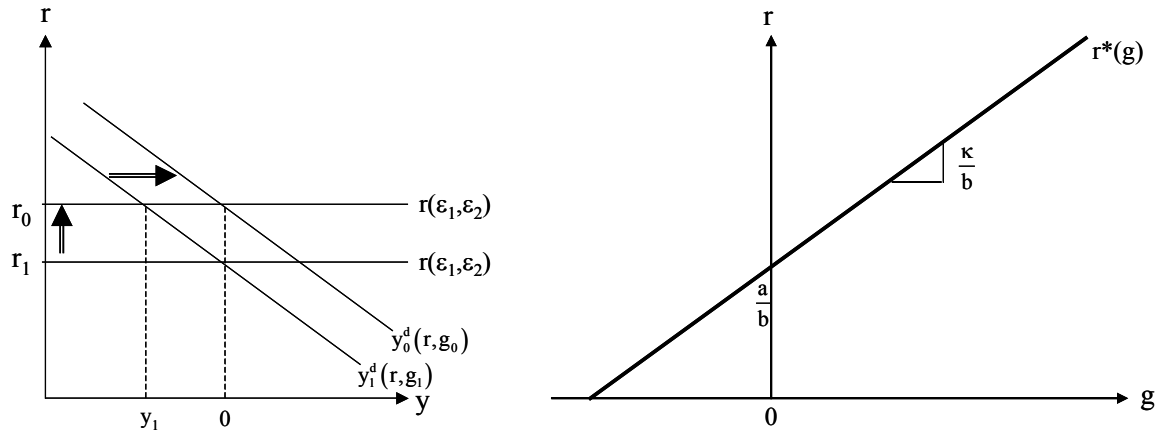
$$(12) \quad L_{CB} = \gamma \left( \pi^e + d(a - br + \kappa g + \varepsilon_1) + \varepsilon_2 - \pi_0 \right)^2 + \lambda (a - br + \kappa g + \varepsilon_1)^2$$

Taking the derivative with respect to the instrument  $r$  and solving it for the first order condition yields:

$$(13) \quad r^{opt} = \frac{a}{b} + \frac{\kappa}{b}g + \frac{1}{b}\varepsilon_1 + \frac{d\gamma}{b(d^2\gamma + \lambda)}\varepsilon_2$$

The reaction function of the central bank specifies the optimal interest rate  $r$  if the government ‘plays’  $g$ . In other words it depicts the optimal response of the central bank to the current stance of fiscal policy  $g$ . If we are initially in a situation were  $\varepsilon_1 = \varepsilon_2 = 0$  and  $r = (a/b)$  an increase in government expenditure from  $g_0$  to  $g_1$  will provoke the central bank to raise real interest rates by  $(\partial r / \partial g) = \kappa/b$  to  $r_1$  (see Chart 3).

**Chart 3: Reaction function of the central bank**



The reaction to supply and demand shocks is unaltered compared to a situation where monetary policy is the only macroeconomic agent (see (Bofinger, Mayer, and Wollmershäuser 2002)). This underlines that monetary policy is able to push its preferred bliss point through.

### 3.2 The reaction function of the government: analytically and graphically

Substituting the IS-equation into the loss function  $L_G$  gives the following optimisation problem for the government:

$$(14) \quad L_G = \beta(a - br + \kappa g + \varepsilon_1)^2 + \varphi g^2$$

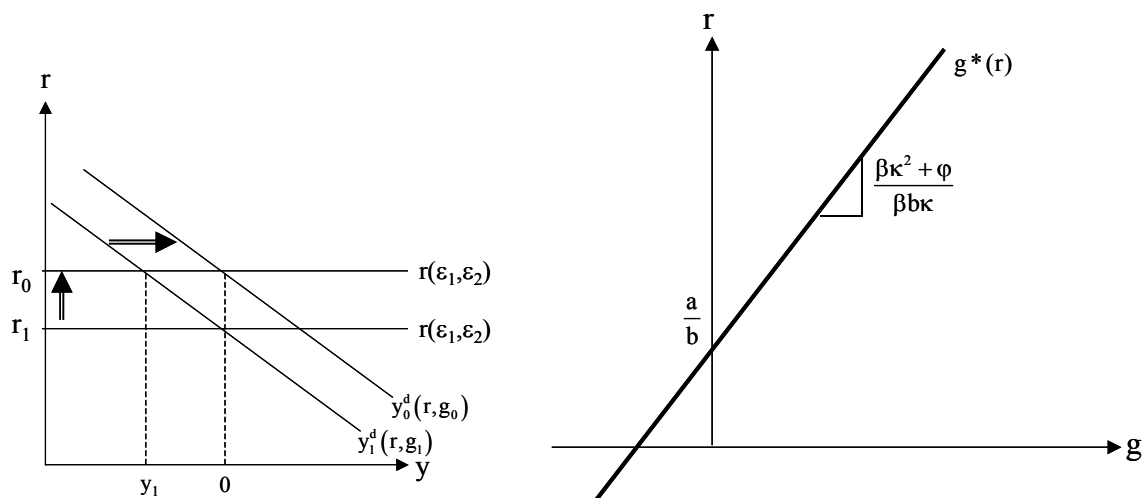
We compute the reaction function by taking the derivative of  $L_G$  with respect to the instrument  $g$ . The reaction function of fiscal policy is then given by:

$$(15) \quad g = -\frac{\kappa\beta a}{\kappa^2\beta + \varphi} + \frac{\kappa\beta b}{\kappa^2\beta + \varphi}r - \frac{\kappa\beta}{\kappa^2\beta + \varphi}\varepsilon_1$$

It depicts the optimal reaction of the government to the current stance of monetary policy. The equation is characterised by the following characteristics:

- The partial derivative of  $g$  with respect to  $r$  is  $(\partial g/\partial r) = (\kappa b/(\kappa^2 + \varphi)) > 0$ . Hence if monetary policy gets more restrictive the political party in power will switch to a more expansionary stance of fiscal policy.
- The higher the weight on output stabilization ( $\beta$ ) the stronger will be the strategic interaction between the government and the central bank.
- The higher the weight on stabilizing the fiscal instrument ( $\varphi$ ) the lower will be the strategic interaction between the two macroeconomic agents.
- Given its objective function (6) fiscal policy only reacts to demand shocks. Following e.g. a negative demand shock  $\varepsilon_1$  fiscal policy will become more expansionary. Note that in contrast to monetary policy the government does not face a lower bound. Hence  $g$  will become negative if  $\varepsilon_1 > 0$ .

**Chart 4: The reaction function of fiscal authorities**



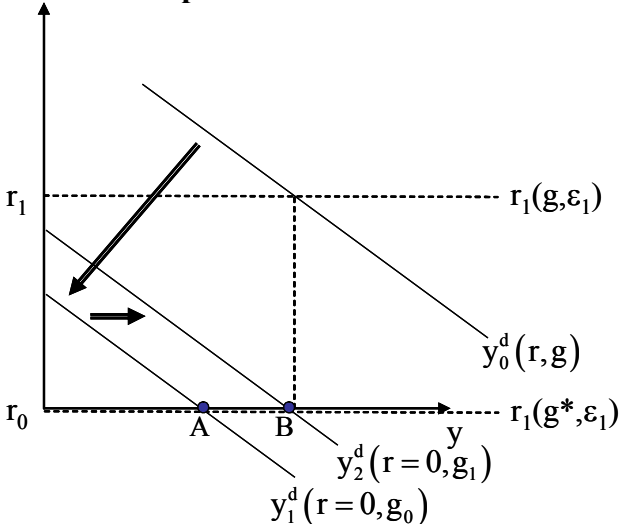
## 4 Application I: Nominal interest rate trap and investment trap as prime examples for a coordinated macroeconomic interaction

In the following we will discuss the nominal interest rate trap and the investment trap as two prime examples where a coordinated action of fiscal and monetary policy is a necessary precondition to restore full employment.

### 4.1 A nominal interest rate trap

A nominal interest rate trap is caused by a large demand shock  $\varepsilon_1$ , e.g. a massive crash in equity markets as we have observed it in Japan over the course of the 1990's or more recently in America and Europe. Obviously even if monetary policy lowers interest rates from  $r_0$  to  $r_1$  it will only be able to partially stabilize economic activity at  $y_1 < 0$  (see Chart 5). Under such a scenario only a coordinated action of both macroeconomic agents is able to restore the bliss point ( $\pi = \pi_0; y = 0$ ). This can be achieved if fiscal policy boosts net expenditures from  $g_0$  to  $g_1$ . This leads to a shift of the aggregate demand curve from  $y_1^d(r=0, g)$  to  $y_2^d(r=0, g^*)$ . The new equilibrium outcome will be point B. The joint stabilization effort restores an output gap of null.

**Chart 5: A nominal interest rate trap**



Algebraically we can describe the macroeconomic interaction as follows. As in the previous sections we assume that the conduct of the central bank and the government is described by Nash behaviour. The zero lower bound becomes binding if  $\varepsilon_1 \leq -a - \kappa g$ . This can be seen as follows:

For ‘normal’ *demand shocks* the behaviour of the central bank is governed by the following reaction function.

$$(16) \quad r = \frac{a}{b} + \frac{\kappa}{b}g + \frac{1}{b}\varepsilon_1$$

If the economy is hit by a large demand shock the central bank can at best-under a deflationary scenario- set the real interest rate equal to zero ( $r=0$ ).

$$(17) \quad 0 = \frac{a}{b} + \frac{\kappa}{b}g + \frac{1}{b}\varepsilon_1$$

Obviously the zero bound will become binding if  $\varepsilon_1 \leq -a - \kappa g$ . Then monetary policy can no longer on its own restore the bliss point (see Chart 5). Now fiscal policy needs to raise public expenditure or cut taxes ( $g \gg 0$ ). Depending on its specific loss function the optimisation problem of the government can be described as follows.

$$(18) \quad \begin{aligned} L_G &= \beta y^2 \\ \text{s.t. } r &= 0 \end{aligned}$$

Inserting the IS-equation yields:

$$(19) \quad L_G = \beta \left( (a + \kappa g) + \varepsilon_1 \right)^2$$

Under a deflationary scenario the reaction function of the government is given by:

$$(20) \quad g = -\frac{\kappa}{a} - \frac{1}{\kappa}\varepsilon_1$$

In response to a negative demand shock it will raise expenditures by  $(1/\kappa)$ . This combined stabilization effort restores the bliss point. This can be shown by inserting the real interest rate and the fiscal stance parameter into the IS-equation:

$$(21) \quad y = a + \underbrace{b\pi_0}_{\text{monetary policy}} + \underbrace{\kappa \left( -\frac{a}{\kappa} - \frac{1}{\kappa} \varepsilon_1 \right)}_{\text{fiscal policy}} + \varepsilon_1 = 0$$

Accordingly, output will be at its potential and the inflation rate will be equal to the inflation target. This shows that at least in theory the two macroeconomic ‘demand side agents’ can protect the economy totally from demand shocks of arbitrary size.

### Box 1: Nominal Interest Rate Trap: Nominal Rates

Let us now assume for reasons of convenience that the instrument of monetary policy is the nominal interest rate. The central bank solves then the following optimisation problem:

$$L_{CB} = (\pi - \pi_0)^2 + \lambda y^2$$

Subject to the following structural equations:

$$\pi = \pi_0 + dy + \varepsilon_2$$

$$y = a - b(i - \pi) + \kappa g + \varepsilon_1$$

Solving the optimisation problem of the central bank leads to the following reaction function:

$$i = \frac{a}{b} + \pi_0 + \frac{1}{b} \varepsilon_1 + \frac{\kappa}{b} g + \frac{b\lambda + d}{b(d^2 + \lambda)} \varepsilon_2$$

As we assume that the economy is hit by a large demand shock the zero lower bound is characterized by the following interest rate equation.

$$0 = \frac{a}{b} + \pi_0 + \frac{1}{b} \varepsilon_1 + \frac{\kappa}{b} g$$

So we can show that the zero lower bound will become binding if:

$$\varepsilon_1 > -(a + b\pi_0 + \kappa g)$$

Under such a scenario fiscal authorities solve the following optimisation problem:

$$L_G = \beta (a + b(i - \pi) + \kappa g + \varepsilon_1)^2$$

The zero lower bound becomes binding if:  $i = 0$ . Under such a deflationary scenario the reaction function of fiscal policy will be given by:

$$g = -\frac{a}{\kappa} - \frac{b}{\kappa} \pi_0 - \frac{1}{\kappa} \varepsilon_1$$

Note as monetary policy is credible we could substitute  $\pi$  by  $\pi_0$ .

We can show that this combined stabilization effort restores the bliss point by inserting the instruments into the IS-equation:

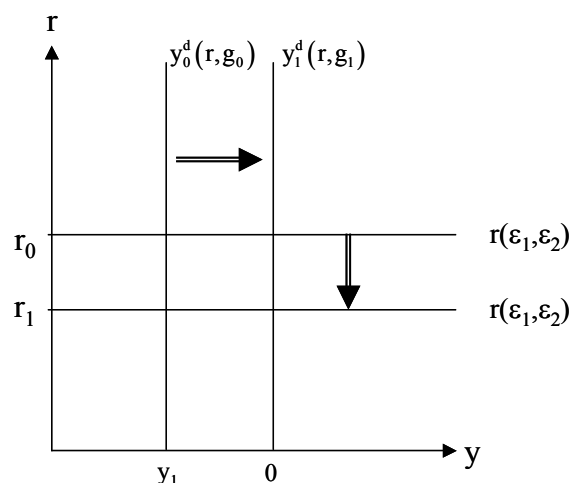
$$y = a - b(-\pi_0) + \kappa \left( -\frac{a}{\kappa} - \frac{b}{\kappa} \pi_0 - \frac{1}{\kappa} \varepsilon_1 \right) + \varepsilon_1$$

Simplifying this expression we see that the output gap will be equal to null. Hence the demand shock is totally stabilized.

## 4.2 Investment Trap

As a second classical case to consider we analyse the investment trap. If the economy is stuck in an investment trap fiscal policy remains the only stabilizing factor. Following Keynes (1936) we assume that investors are ad most pessimistic over the course of economic activity. As a consequence investment activity does not depend on interest rates. Accordingly the central bank looses its impact on investment and on aggregate demand (see Chart 6). If we are initially in  $y_1$  an interest rate cut will not stimulate economic activity. Only a more expansionary fiscal stance ( $g \gg 0$ ) which shifts the initial demand equation from  $y_0^d(r, g_0)$  to  $y_1^d(r, g_1)$  can stabilize output at its potential.

**Chart 6 An Investment Trap**



Algebraically we can describe such a scenario if we set the interest rate elasticity  $b$  in the aggregate demand equation (2) equal to null.

$$(22) \quad y = a + \kappa g + \varepsilon_1$$

The overall policy outcome under such a scenario depends on the loss function of the government. Table 1 depicts a battery of possible deterministic solutions. For the sake of exposition we will assume that fiscal policy is described by:

$$(23) \quad L_G = \beta y^2$$

Then the first order condition will be given as:

$$(24) \quad g = -\frac{a}{\kappa} - \frac{1}{\kappa} \varepsilon_1$$

If fiscal policy is conducted according to equation (24) output will be equal to null.

$$(25) \quad y = 0$$

In summary, the two proceeding sections demonstrate that the BMW framework can easily accommodate investment traps and the nominal interest rate trap, which are prime examples for the necessity of fiscal policy in standard intermediate textbooks.

**Table 1 Comparing the outcomes under different assumptions on the loss function**

|                      | Fiscal Policy Outcomes in an Investment trap |   |  |  |
|----------------------|--|---|--|--|
|                      | No Stabilization Bias                        |   | Stabilization bias                         |  |
|                      | $L_G = \beta y^2$                            | $L_G = \beta y^2 + \phi g^2$                        | $L_G = \beta(y - k)^2$                     | $L_G = \beta(y - k)^2 + \phi g^2$  |
| <b>Fiscal Policy</b> | $g = -\frac{a}{\kappa}$                      | $g = -\frac{\beta a \kappa}{\beta \kappa^2 + \phi}$ | $g = -\frac{a}{\kappa} + \frac{k}{\kappa}$ | $g = -\frac{\beta \kappa a}{\beta \kappa^2 + \phi} + \frac{\beta \kappa k}{\beta \kappa^2 + \phi}$ |

## 5 Application II: A clear assignment on the target level does not lead to a clear assignment on the instrument level

In some economic quarters it is conventional wisdom that macroeconomic targets should be distributed as follows on the individual macroeconomic agents:

- Monetary policy is responsible for price stability, which will be further interpreted as keeping the inflation rate near the inflation target.
- Fiscal policy is responsible for economic growth, which will in the following be interpreted as keeping output near its potential.

This view of macroeconomic job sharing was for instance stated by the German “Sachverständigenrat zur Begutachtung der gesamtwirtschaftlichen Lage” in 1978 among others. In the following we will show that this macroeconomic assignment does not lead to a unique Nash equilibrium. In other words the simple idea ‘one macroeconomic player per macroeconomic goal’ leads to multiple equilibrium, which shows that the clear assignment on the target level does not translate into a clear assignment on the instrument level.

If monetary policy is solely responsible for price stability the loss function can be stated as follows:

$$(26) \quad L_{CB} = (\pi - \pi_0)^2$$

Hence, the central bank is only interested in keeping the inflation rate near the inflation target. Output does not play an independent role as a goal variable for the conduct of monetary policy. Following (Svensson 1997) we may call this preference type an inflation nutter.

As a second optimizing agent we introduce fiscal policy. Fiscal policy has the only assignment to keep output at its potential. Accordingly we can state the loss function as follows:

$$(27) \quad L_G = y^2$$

In order to evaluate the outcome of the game under this assignment we have to compute the reaction functions of fiscal and monetary authorities.

The central bank solves the following optimization problem:

$$(28) \quad L_{CB} = (da - dbr + d\kappa g)^2$$

which translates into the following first order condition:

$$(29) \quad \frac{\partial L_{CB}}{\partial r} = 2(da - dbr + d\kappa g)(-db) = 0$$

Rearranging and solving for the instrument  $r$  we arrive at the following reaction function for monetary policy:

$$(30) \quad r = \frac{a}{b} + \frac{\kappa}{b} g$$

The optimization problem of fiscal authorities is described by the following optimization problem.

$$(31) \quad L_G = y^2$$

Inserting the IS-equation in  $L_G$  yields:

$$(32) \quad L_G = (a - br + \kappa g)^2$$

which gives the following optimization problem:

$$(33) \quad \frac{\partial L_G}{\partial g} = 2(a - br + \kappa g)\kappa = 0$$

The reaction function of fiscal policy is given by:

$$(34) \quad g = -\frac{a}{\kappa} + \frac{b}{\kappa}r$$

Obviously the Nash equilibrium is not uniquely determined as the two reaction functions are identical. In other words we have two variables to be determined  $r$  and  $g$ , but only one equation. Hence by simple calculus we showed that a clear assignment among fiscal and monetary authorities does not generate a unique Nash equilibrium. Therefore the simple wisdom of macroeconomic job sharing saying that each agent should only be responsible for one goal does not settle the issue of the optimal instrument mix uniquely.

## **6 Application III: Modelling the strategic interaction between the central bank and the government in ordinary times**

Application I dealt with the classical cases where monetary and fiscal policy can only in a combined effort restore the bliss point. Of course these cases, also very much discussed in economic textbooks are only of minor importance in ordinary times. In other words a great depression does not come around the corner every year. Therefore we need to describe more likely scenarios where neither monetary nor fiscal policy faces a binding restriction such as the zero lower bound on its instrument. Hence in the following section we will model the strategic interaction between fiscal and monetary policy based on the notion of a Nash equilibrium in ordinary times.

### **6.1 The Nash equilibrium: analytically and graphically**

Algebraically we can derive the reduced form solution by making use of the reaction function of fiscal and monetary policy. Then we are equipped with two equations and two unknowns which allows potentially for a unique (stable) equilibrium. Solving this system of equations (equation (13) and (15)) leads to the following reduced form of the system:

$$(35) \quad r = \frac{a}{b} + \frac{1}{b} \varepsilon_1 + \frac{d(\kappa^2 + \varphi)}{b(d^2 + \lambda)\varphi} \varepsilon_2$$

$$(36) \quad g = \frac{d\kappa\beta\gamma}{d^2\gamma\varphi + \lambda\varphi} \varepsilon_2$$

In the absence of macroeconomic shocks ( $\varepsilon_1 = \varepsilon_2$ ) not surprisingly the central bank sets the interest rate equal to its long run equilibrium value and the fiscal stance parameter will be equal to null ( $g = 0$ ). With equations (35) and (36) at hand we can easily compute the reduced form expression for the output and inflation gap.

$$(37) \quad y = -\frac{d\gamma}{d^2\gamma + \lambda} \varepsilon_2$$

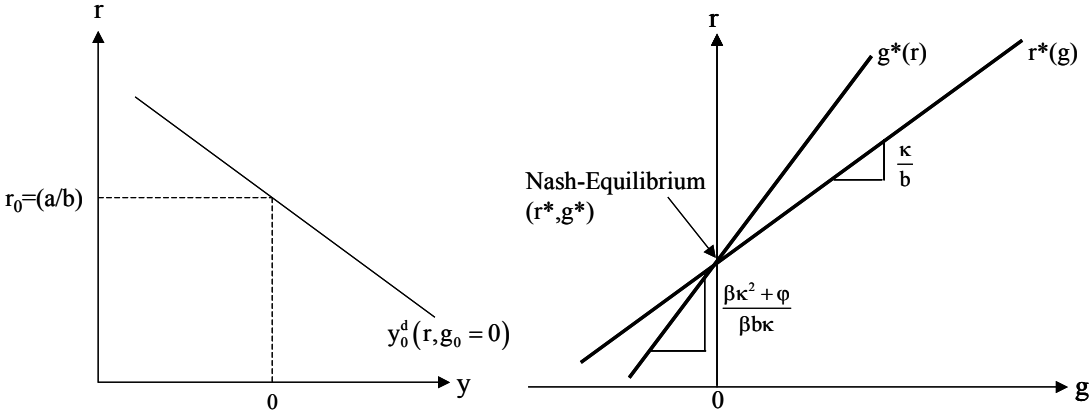
$$(38) \quad \pi = \pi_0 + \frac{\lambda}{d^2\gamma + \lambda} \varepsilon_2$$

Obviously this is the preferred outcome for each macroeconomic player as it restores the bliss point; the output gap will be equal to null and the inflation rate will be equal to the inflation target in the absence of supply shocks. The results share some very interesting features. In particular one can see that the reduced form expression for  $g$  does not depend on the demand shock  $\varepsilon_1$ . Accordingly the net impact of fiscal policy following a demand shock is zero. The initial expansionary stance of fiscal policy ( $g > 0$ ) is wiped out by the second and third round effects of higher real interest rates. As the reaction function of monetary policy tells us the increase in fiscal spending leads to a more contractionary monetary stance which will be counteractive for the initial fiscal expansion.

The unique Nash-equilibrium is found graphically as the intersection of the two reaction functions in the  $(g,r)$ -space. Under reasonable parameterisation the slope of the  $g^*(r)$  function

will be steeper than the slope of the  $r^*(g)$ -function which guarantees a unique and stable Nash equilibrium.<sup>2</sup>

**Chart 7 Nash equilibrium in a game without shocks\***



If we translate this result into the usual  $(y;r)$ -space we see that in the absence of macroeconomic shocks the bliss point can easily be achieved.

**6.2 Comparative static analysis of the optimal solution**

What are the consequences for the conduct of monetary policy if the government becomes more liberal (increasing  $\beta$ )? Does optimal fiscal policy change if the central bank becomes more conservative (increasing  $\gamma$ )? To answer such questions we evaluate the reduced form solution of the game in depth. In particular we evaluate equations (35) and (36) with respect to the preference parameters of fiscal  $(\beta, \varphi)$  and monetary  $(\gamma, \lambda)$  authorities<sup>3</sup>.

As equation (39) shows an increasing preference of fiscal policy to stabilize  $g$  (increasing  $\varphi$ ) leads ceteris paribus to a decrease in the real interest rate. If fiscal policy gets on average less anticyclical monetary policy will step in to stabilize the output gap more actively in response to supply shocks.

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<sup>2</sup> E.g.,  $a=1.2, \beta=0.9; \phi = \kappa = \varphi = 1; b = 0,4$  ; Inserting these values yields the following concrete reaction functions for *fiscal policy*:  $r = 3 + 5.28g$  ; and *monetary policy*:  $r = 3 + 2,5g$  .  
<sup>3</sup> For a similar exercise in a Barro-Gordon framework see (Demertzis 1999).

$$(39) \quad \frac{\partial r}{\partial \varphi} = -\frac{d\kappa^2 \beta \gamma \varepsilon_2}{b(d^2 \gamma + \lambda) \varphi^2} \leq 0$$

If fiscal authorities put a higher weight on stabilizing the output gap (increasing  $\beta$ ) the monetary stance will become tighter. Hence one may conclude that more liberal governments induce c.p. a more conservative central bank.

$$(40) \quad \frac{\partial r}{\partial \beta} = \frac{d\kappa^2 \gamma \varepsilon_2}{bd^2 \gamma \varphi + b\lambda \varphi} \geq 0$$

Equation (41) displays that an increasing weight on stabilizing the inflation rate around the inflation target (increasing  $\gamma$ ) leads to more anticyclical fiscal policy in response to supply shocks. Hence if monetary policy is conducted in a more conservative fashion fiscal policy tends to become more liberal.

$$(41) \quad \frac{\partial g}{\partial \gamma} = \frac{d\kappa \beta \lambda \varepsilon_2}{(d^2 \gamma + \lambda)^2 \varphi} \geq 0$$

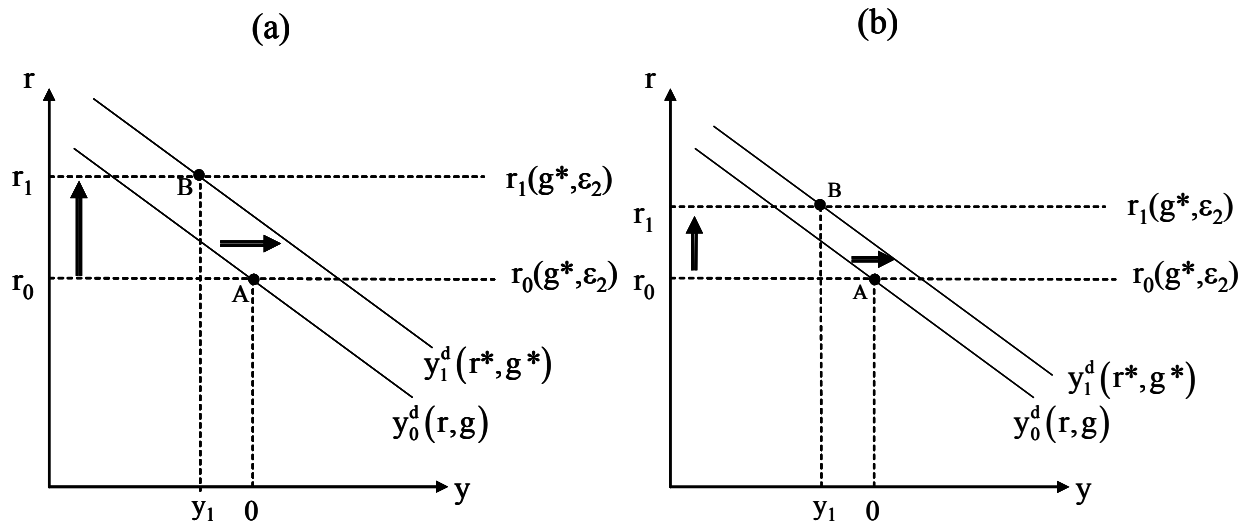
Equation (42) displays that an increasing weight of monetary policy towards stabilizing the output gap leads to a less active fiscal policy. Hence output stabilization done by monetary authorities versus output stabilization conducted by fiscal authorities are substitutes in nature (see Chart 8).

$$(42) \quad \frac{\partial g}{\partial \lambda} = -\frac{d\kappa \beta \gamma \varepsilon_2}{(d^2 \gamma + \lambda)^2 \varphi} \leq 0$$

In summary the partial derivatives (39)-(42) show that identical macroeconomic outcomes in response to supply shocks on the goal level can be implemented by an (infinite) amount of instrument combinations on the instrument level.

The concrete macroeconomic job sharing depends on preferences. Panel (a) depicts a scenario where we have a more conservative central bank and accordingly a more liberal government. Panel (b) characterizes a situation in which we have a more conservative government. Accordingly the reaction of monetary policy in response to supply shocks will be smaller.

**Chart 8 Supply shocks in the game between fiscal and monetary authorities**



### 7 Application III: An overly ambitious central bank

In the Barro/Gordon literature (Barro 1983) it is common practice to introduce an inflationary bias on the side of monetary authorities. The usual argument is that there are monopolistic distortions in good and labour markets so that monetary policy steps in to implement a globally optimal solution by trying to relax monetary conditions on average. Nevertheless an increasing number of economists take the point of view that the ‘inflationary bias’ is largely spurious<sup>4</sup>. It is a stylised fact that most governments have made excessive deficit spending over the last decades which has resulted in an increasing debt burden. Therefore we hold it plausible to assume that there is a ‘stabilization bias’ present in the behaviour of fiscal authorities which induces a permanent deficit spending ( $g > 0$  in equilibrium). We model this stabilization bias by introducing a  $k$ -factor in the loss function.

$$(43) \quad L_G = \beta(y - k)^2 + \varphi g^2$$

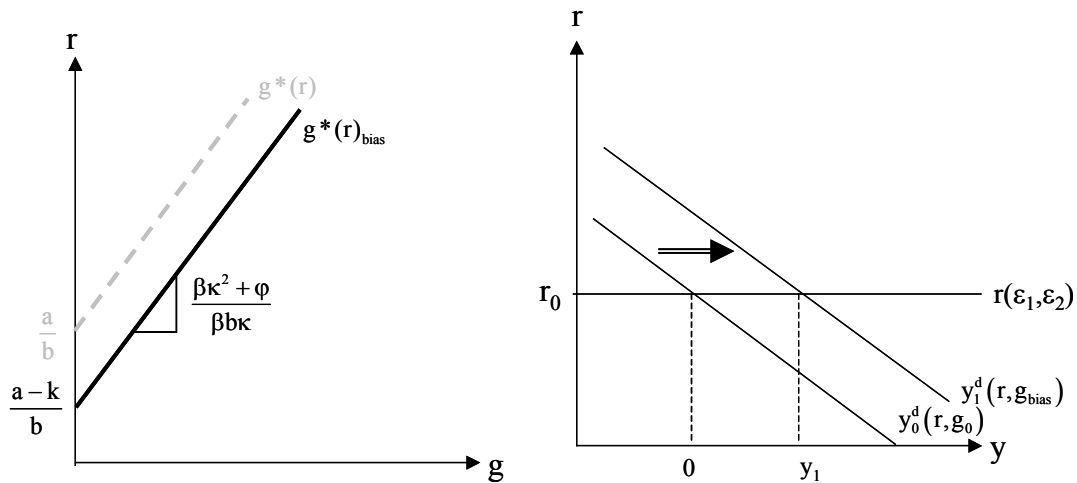
The reaction function of fiscal policy will then be given by:

<sup>4</sup> See for instance (Blinder 1998).

$$(44) \quad g = \frac{\beta\kappa k}{\beta\kappa^2 + \varphi} - \frac{\beta\kappa a}{\beta\kappa^2 + \varphi} + \frac{\beta b\kappa}{\beta\kappa^2 + \varphi} r$$

Equation (44) shows that the introduction of a stabilization bias leads on average to a more expansionary fiscal stance by a factor of  $(\beta\kappa k / \beta\kappa^2 + \varphi)$ . Graphically this translates into a downward shift of the reaction function of fiscal policy in the  $(r;g)$ -space. Hence for any given level of interest rates  $r$  fiscal policy will be more expansionary on average.

**Chart 9: The reaction function of the government with an inflationary bias**



The reaction function of monetary policy will still be given by:

$$(45) \quad r = \frac{a}{b} + \frac{\kappa}{b} g,$$

As Chart 10 shows in a scenario where a ‘stabilization bias’ is present the Nash-equilibrium will be shifted towards the Northeast. In other words fiscal policy will be on average more expansionary whereas monetary policy will be more restrictive.

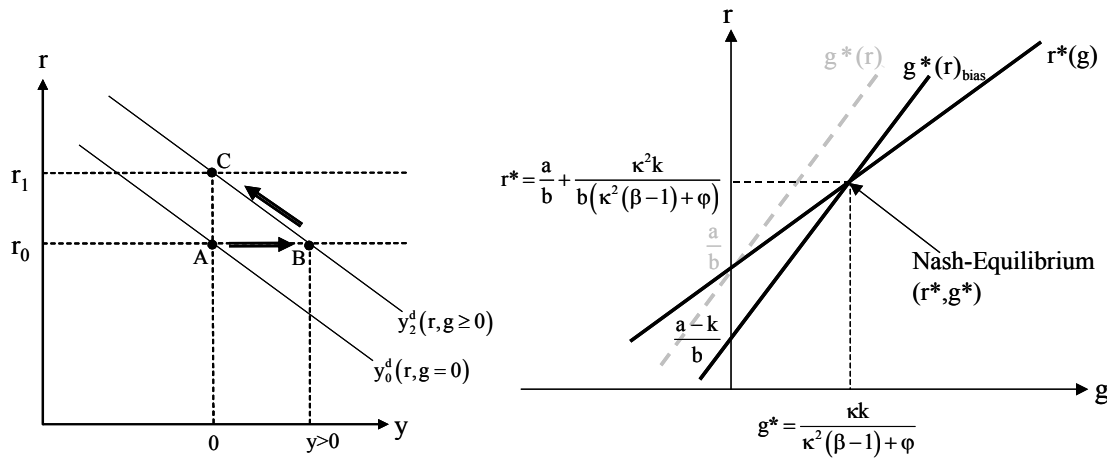
Algebraically we can derive the Nash equilibrium by inserting equation (44) into equation (45). Solving for  $g$  we get the following deterministic reduced form outcome:

$$(46) \quad g = \frac{\kappa k}{\kappa^2 (\beta - 1) + \varphi}$$

Due to the stabilization bias fiscal policy will on average display a negative general government balance. Hence equation (46) nests some unpleasant arithmetic's, which is not sustainable in the long run. Inserting equation (46) into equation (45) yields the following reduced form solution for the interest rate.

$$(47) \quad r = \frac{a}{b} + \frac{\kappa^2 k}{b(\kappa^2(\beta-1) + \varphi)}$$

**Chart 10: Nash equilibrium with stabilization bias**



If we translate these results into the  $(y, r)$ -space we see that the overall macroeconomic outcome on the goal level will be identical ( $y=0$ ) to a scenario without stabilization bias (see Chart 10). The new equilibrium is point C. In response to the more expansionary fiscal policy monetary policy raises real interest rates from  $r_0$  to  $r_1$ . Hence the reduced form of the output gap and the inflation rate will be given by equation (48) and (49).

$$(48) \quad y = 0$$

$$(49) \quad \pi = \pi_0$$

In summary the central bank unwinds the expansionary effects induced by the stabilization bias. Thereby the final outcome of the game will be the bliss point of the central bank. Equations (48) and (49) impressively demonstrate that monetary policy is the dominant macroeconomic player on the demand side as the use of its instrument does not produce any costs. In the appendix we

show that the situation is reversed when the central bank has interest rate smoothing as an independent goal in its loss function and fiscal policy can use its instrument at zero cost.

## **8 Conclusions**

In this paper we modelled the strategic interaction between fiscal and monetary authorities under three different scenarios within the BMW framework. We showed that in ordinary times monetary policy is able to reach its preferred bliss point as fiscal policy uses its instrument only cautiously. Nevertheless in a deflationary economic environment when monetary policy hits the zero lower bound ( $r=0$ ) and is unable to protect the economy from shocks we also need fiscal policy as demand management agent in order to squeeze cyclical unemployment. Additionally we demonstrated that an overly ambitious political party in power might be responsible for an unsustainable fiscal policy.

## APPENDIX

### Interest rate smoothing

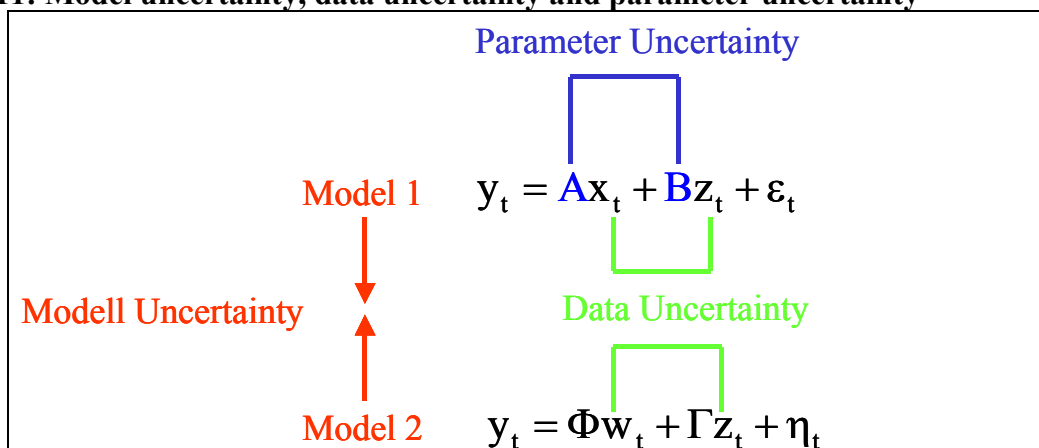
So far we have implicitly assumed that monetary policy can push its targets through as it has a structural advantage over fiscal policy as it does not face any costs in terms of using its instrument. We will shortly prove this assumption by showing that an unrestricted fiscal agent is able to push its target through compared to a restricted central bank. Note that we do not think that this scenario is realistic but it serves as a useful experiment to state the case previously made. The scenario is calculated by using the following equations:

$$(50) \quad L_{CB} = \gamma(\pi - \pi_0)^2 + \psi \left( r - \frac{a}{b} \right)^2$$

Besides stabilizing squared deviations of the inflation rate around the inflation target the central bank cares on smoothing the interest rate around its equilibrium value of  $r = a/b$ .

In other words the central bank tries to stabilize the real short term interest rate around its long run equilibrium value which is given by  $(a/b)$ . Interest rate smoothing can be rationalized by the following list of arguments. The main decision making body of a central bank faces several kinds of uncertainties while taking its decision on interest rates. Among them are model uncertainty, data uncertainty and Brainard uncertainty.

**Chart 11: Model uncertainty, data uncertainty and parameter uncertainty**



It is well documented that each type of these uncertainties tends to reduce the aggressiveness with which policymakers set their instruments in response to shocks that hit the economy.

- Goodhart (1996) pointed out that central bank's try to avoid interest rate changes in the near future that reverse current interest rate decisions as those might be interpreted by the public as a withdrawal of a previous error. Therefore so Goodhart, central bank's tend not to change interest rates until evidence of shifts in inflationary pressure occur.
- Rudebusch (2002) has shown that interest rate smoothing can be a direct consequence of auto correlated shocks as it is plausible to assume that interest rates exhibit a comovement with these shocks.
- Additionally Brainard uncertainty may be at the centre of a gradual interest rate policy. It can be shown that uncertainty on individual parameters of the true economic model induces a more gradual interest rate setting behaviour on behalf of the central bank (Carl E. Walsh 2003).
- As mentioned by Woodford in particular in an environment with forward looking agents interest rate smoothing serves as an instrument to make use of peoples expectations on the stabilizing properties of monetary policy itself. If monetary policy is implemented gradually economic agents will expect further interest rate steps in the same direction which will for instance prevent some price setters from increasing their prices in the vague of a supply shock. Thereby the central bank can improve its performance in terms of lower variances of target deviations (Woodford 1999).

Fiscal policy is guided by the loss function (51). Hence we assume that the political party in power has a stabilization bias, which might be motivated by the desire to boost the propensity to be re-elected.

$$(51) \quad L_G = \beta(y - k)^2$$

Solving the model we arrive at the following reaction functions for fiscal and monetary policy respectively:

$$(52) \quad r = \frac{a}{b} + \frac{bd^2\gamma\kappa}{b^2d^2\gamma + \psi} g$$

$$(53) \quad g = \frac{k}{\kappa} - \frac{a}{\kappa} + \frac{b}{\kappa} r$$

The reduced forms are given by:

$$(54) \quad r = \frac{a}{b} + \frac{bd^2k\gamma}{\psi}$$

$$(55) \quad g = \frac{k(b^2d^2\gamma + \psi)}{\kappa\psi}$$

Which translates into the following macroeconomic outcomes:

$$(56) \quad y = k$$

$$(57) \quad \pi = \pi_0 + dk$$

Equations (56) and (57) impressively demonstrate that the dominance of monetary policy that prevailed in the previous sections stems from the fact that it does not face any costs while using its instrument. Whereas fiscal policy needs to back up its expenditures by revenues, monetary policy can freely set its instrument without any material restrictions.

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